A Note on the Analytics of Aerial Triangulation with GPS Aerial Control

Abstract

Aerial triangulation with GPS control in the aircraft, and almost no ground control, is about to become operational: the key technology behind this is GPS kinematic positioning. The new "control" information is introduced as observational data into the photogrammetric network in a combined block adjustment. This paper develops some aspects of the analytics of combined block adjustment from an operational point of view. This is accomplished in connection with the integration of GPS blocks into existing control networks for a twofold purpose: employing existing information and further exploiting GPS aerial triangulation results.

Introduction

The term "aerial control point" is defined here as the position of a point in a survey aircraft at the exposure moment of an image. In a more general context, aerial triangulation with aerial control was already undertaken in the late sixties when the MUSAT program, developed by A. Elsass, was used to process data from the Lunar Orbiter missions. According to Slama (1980): "This was also the first time a triangulation project had utilized a computer program with orbital constraints to establish control for mapping."

Compared to all the research and development reported in Slama (1980) — a chapter of truly delightful reading on satellite photogrammetry — in GPS aerial triangulation, once the coordinates of aerial control points have been computed from GPS pseudorange or phase observations, their integration into a bundle adjustment is a minor: mathematical and software problem, regardless of whether or not they are affected by some systematic errors.

Aerial control points are determined with kinematic (differential) GPS techniques. With the exception of small scale photography where pseudoranges can carry out the task, phase observations are used to perform precise (<10 cm) kinematic positioning. Over the past five years many experiments have been conducted which have validated the concept and have shown its attainable accuracy. The experiments also had to overcome a number of difficulties related to state-of-the-art kinematic GPS, among others, the correct determination of integer ambiguities in non-static applications. The result of these experiments and related research and development efforts is a well-defined procedure for the collection, analysis, and processing of GPS phase observations, as well as the integration of resulting aerial control points into a photogrammetric block adjustment.

According to Ackermann (1992a), GPS aerial triangulation encounters a number of problems: (a) the GPS antenna offset, (b) the camera time offset, (c) GPS initial phase ambiguities, (d) GPS signal interruptions, and (e) the datum problem.

As already mentioned, over the last five years a methodology has been developed which overcomes these technical difficulties, especially those related to the more critical aspects (c) and (d), which is based on the a posteriori modeling — in the block adjustment — of possible mismodeled parameters in GPS phase processing, mainly the incorrect determination of integer ambiguities (Friese, 1990).

Credit for the development of such an operational methodology goes mainly to the group from the Institute for Photogrammetry, University of Stuttgart (Ackermann, 1992a; Ackermann, 1992b; Friese, 1990; Schade, 1992). Thanks to the "Stuttgart approach," practical application of GPS-supported aerial triangulation did not have to await the unilateral solution of the problems previously mentioned. The "Stuttgart approach" is global in that GPS techniques and photogrammetric block adjustment techniques cooperate in solving the point determination and sensor orientation problem.

The standpoint of this paper is the above approach and also the need to integrate GPS aerial triangulation into everyday photogrammetric and mapping projects; it will discuss the analytic aspects of GPS aerial triangulation in relation to problems (a) and particularly (e). (Although a question which should not be forgotten (Kubik, 1992), problem (b) will not be discussed because, with the help of a GPS-aided photogrammetric navigation system, it is always possible to trigger the camera shutter at or close to a GPS observation epoch, thus practically solving the aircraft’s trajectory interpolation problem.)

Development of the Model

The following notation conventions will be used throughout the paper: matrices and vectors will be written in bold type, $A^T$ will denote the transpose of $A$, and if $A = (c)$, then $\mu A = (\mu c)$, unless otherwise stated. A list of the mathematical symbols used is given in Table 1.

The integration of kinematic GPS aerial control into photogrammetric networks is often referred to as the integration of "directly observed camera orientation data." This designation is correct because this type of data is so closely related to the camera orientation parameters that for some small scale applications aerial control can be regarded as actual orientation data. From a mathematical point of view, this means that the assumed functional model is

$$x^i = X^i,$$  

where $X^i = (X^i, Y^i, Z^i)$ are the observed coordinates of the position part of the orientation elements and where $X^i = (X^i, Y^i, Z^i)$ are the position unknowns of the $i$ image orientation parameters. Although the very first simulation studies were based on Equation 1, in practice the situation is more

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TABLE 1. LIST OF MATHEMATICAL SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>x_i</td>
<td>GPS aerial control observation for image j</td>
</tr>
<tr>
<td>X_i</td>
<td>Projection coordinate of image j</td>
</tr>
<tr>
<td>X_e</td>
<td>Eccentricity vector of the antenna center in the j reference system</td>
</tr>
<tr>
<td>R_i</td>
<td>Rotation matrix of the j image</td>
</tr>
<tr>
<td>X_1</td>
<td>Origin shift, scale factor and rotation matrix of a seven-parameter transformation</td>
</tr>
<tr>
<td>X_i, V_i</td>
<td>Constant and &quot;velocity&quot; terms of a linear drift parameter set</td>
</tr>
<tr>
<td>t_i</td>
<td>Time of exposure of image j</td>
</tr>
<tr>
<td>t_o</td>
<td>Time origin of the s drift parameter set</td>
</tr>
<tr>
<td>X_i</td>
<td>Ground control observation at point j</td>
</tr>
<tr>
<td>X_p</td>
<td>Ground point j</td>
</tr>
<tr>
<td>X_p, X_p, i</td>
<td>Biases of interior orientation parameters (E,N,H)</td>
</tr>
<tr>
<td>[c, b, h]</td>
<td>Geodetic coordinates (E,N,H)</td>
</tr>
<tr>
<td>X_p(l, c, h)</td>
<td>Projection center geocentric coordinates parameterized by (l, c)</td>
</tr>
<tr>
<td>M(l, c, b)</td>
<td>Transformation matrix of the L system whose origin is (c, b, h)</td>
</tr>
<tr>
<td>X_p, X_p, i</td>
<td>Ground point geocentric coordinates parameterized by (l, c)</td>
</tr>
<tr>
<td>N</td>
<td>Geodululation</td>
</tr>
<tr>
<td>c_1</td>
<td>Standard error of geodululation</td>
</tr>
<tr>
<td>N</td>
<td>Relative geodululation</td>
</tr>
<tr>
<td>c_2</td>
<td>Standard error of relative geodululation</td>
</tr>
<tr>
<td>G.E.  -&gt;  H.O.</td>
<td>Transformation: geodetic (l, c, h) to local (X,Y,Z) coordinates</td>
</tr>
<tr>
<td>G.E.  -&gt;  P.R.</td>
<td>Transformation: (l, c) to map projection (E,N) coordinates</td>
</tr>
<tr>
<td>D.H.  -&gt;  E.H.</td>
<td>Transformation: orthometric (H) to ellipsoidal heights (h)</td>
</tr>
</tbody>
</table>

complex and a number of other parameters and constants are to be considered. The following definitions are required: S will stand for the satellite reference system, for instance, 

WGS84: L for a local Cartesian reference system, for instance, 

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 resultant of the eccentricity vector of the 

receiver’s antenna center in the j reference system. R_i = r_0_i is the rotation matrix of the j image system (parameterized, for instance, by the usual φ, ω, and κ angles), and X_p = [X_p, Y_p, Z_p] are the j image projection center coordinates in the L reference system, then the coordinates of the receiver antenna in the same local system are

X_p = X + (1 + μ)RX_p.  

Assume that the satellite reference system, S, and the local reference system, L, are related through a general seven-parameter transformation

X_p = X + (1 + μ)RX_p.  

where R is the rotation matrix of the L system with respect to the S system, and (1 + μ) is the scale factor between them and X, the origin shift. Then, for the observed coordinates x_i at the j exposure moment, the following equation holds (see Lucas (1987)):

x_i = X + (1 + μ)R(x_i + R/X_i).  

(4)

if, according to the results in Ackermann (1992a), Col- 

luna (1989), Frič (1990), and Schade (1992), additional linear correction parameters – the so-called drift param-

ters – are introduced, Equation 4 becomes

x_i = X + (1 + μ)R(X_i + X/X_i) + X_i + V_i(t_i - t_o).  

(5)

where X_i = [X_i, Y_i, Z_i]^T, V_i = [V_x, V_y, V_z]^T constitute the s drift parameter set; obviously, X_i and V_i are the constant shift and "velocity" terms, respectively, of the drift parameter set. An image j can only be related to one drift parameter set, i.e., the time at which the j image was taken, and it is reduced to a drift set time origin t_o. The choice of t_o is not critical at all, but it is usually computed as

\[ t_o = \left( \frac{\sum \nu_i}{n} \right) / n, \]

where the n_i images s_i, dots, s belong to the drift set s. If the coordinates of a number of object points in the S system are known, additional observation equations which essentially contribute to the estimation of the datum transfer parameters can be set based on

x_i = X + (1 + μ)RX_i,  

(6)

where X_i = [X_i, Y_i, Z_i]^T are the coordinates of a ground object point in the L system and x_i = [x_i, y_i, z_i]^T are the observed or known coordinates of the same point in the S system.

In fact, Equation 5 represents elementary geometrical relationships (see Figure 1) but relates three observed values to 22 unknowns which are highly correlated (seven datum transformation parameters, six image exterior orientation parameters, three antenna eccentricity vector components, and six drift parameters). This correlation exists, for instance, between datum transfer and drift parameters or between the antenna offset X_i and the orientation elements. At the very least, it would seem that the antenna offset should be measured input data recorded by any hardware means at each exposure moment, so it can be regarded as a constant in Equation 5 or as an observation x_i modeled in the usual way x_i = X_i.

Correlations also exist between the above parameters and possible additional self-calibration parameters (Δφ, Δω, Δκ) of the inner orientation elements, i.e., corrections to camera constant and principal point coordinates. It might well be that small inconsistencies detected in some of the experiments conducted so far are caused by an incorrect knowledge of the above elements during the flight. This fact is again nothing new. So far, however, systematic errors in the exterior orientation parameters produced by (Δφ, Δω, Δκ) biases of the inner orientation elements were harmless because they produced no global inconsistencies. Today this is no longer true because the exterior orientation elements are the link to the GPS aerial control. In the experiments conducted so far, it seems that the drift parameters are playing the role of nuisance parameters, absorbing a number of unmodeled effects.

Note that Equation 5 can also be employed when an array of GPS antennas is used for attitude determination. For each exposure moment x_i, there will be a set of aerial control observations x_i,...,x_i and the corresponding antenna offsets X_i,...,X_i, whereby full 3 × 3 observation covariance matrices are to be considered in the adjustment in order to account for the inner geometric strength of the antenna array.

The next sections deal with three key aspects related to the new aerial control observables: the issue of coordinate transformations, the role of the geod, and the antenna eccentricity vector.
WGS84 and the User System

Equation 5 is the fundamental equation for the integration of aerial control into bundle photogrammetric blocks, where point and orientation parameters are expressed in a local Cartesian coordinate system $L$. There are a number of equivalent approaches to exploit Equation 5 which will be discussed in this section. Depending on the adopted approach, the aerial control observations, the ground control observations, and the adjusted point and orientation parameters will have to undergo different coordinate transformations before and after the adjustment.

Equation 5 itself corresponds to the case where aerial control observations are given in their original form, that is, relative or absolute coordinates in the $S$ system. Then, the "correct" sequence of coordinate transformations is displayed in Figure 2, where some transformation steps may be skipped if the block size is small.

Figure 2 also displays the required transformations if GPS control has already been transformed to a local Cartesian reference system $L$; then, the equation to be used is

$$ x' = X + R(X_d + X_s + V_1(t - t_1)). \quad (7) $$

A further possibility exists which avoids transformations to the auxiliary $L$ system by performing the block adjustment in geodetic coordinates and ellipsoidal heights in the $U$ system. Then, Equation 5 becomes

$$ x' = X + (1 + \mu)R(X(\lambda, \varphi, h) + M(\lambda, \varphi)X_d) + X_1 + V_1(t - t_1). \quad (8) $$

where $X'(\lambda, \varphi, h)$ is the projection center vector of geocentric coordinates parametrized by the geodetic coordinates $(\lambda, \varphi, h)$, and where the transformation matrix $M(\lambda, \varphi)$,

$$ M(\lambda, \varphi) = \begin{pmatrix} -\sin \lambda & -\cos \lambda \sin \varphi & \cos \lambda \cos \varphi \\ \cos \lambda & -\sin \lambda \sin \varphi & \sin \lambda \cos \varphi \\ 0 & \cos \varphi & \sin \varphi \end{pmatrix}. $$

keeps the traditional meaning of the $i$ image orientation angles $\omega, \phi, \kappa$. Of course, in this case the collinearity equations must be derived accordingly by introducing $M(\lambda, \varphi, h)$ in their vector formulation. Similarly, Equation 6 becomes

$$ x_i = X + (1 + \mu)R X(\lambda, \varphi, h). \quad (9) $$

where $X(\lambda, \varphi, h)$ is the $i$ ground object point in the $U$ system. It is noticed that the meaning of the datum transfer parameters is not the same in the above Equations 8 and 9 as in Equations 5 and 6: in Equations 8 and 9, $[X, \mu, R]$ define a transformation between the $S$ and the $U$ systems where $R$ is always close to the unit matrix. A layout of the required coordinate transformations is given in Figure 3.

At the Institut Cartogràfic de Catalunya (ICC), after having used Equations 5 and 7 in several test blocks, we plan on using Equation 8 for routine production tasks. Permanent GPS fiducial stations will be acting as reference stations for kinematic airborne GPS positioning; observations $x'$ will be referred to the WGS84 system; and good approximations for $X, \mu, R$ will be known from previous GPS terrestrial campaigns. The only transformations left for aerial triangulation are those between coordinates $(\lambda, \varphi, h)$ and horizontal map projection coordinates and orthometric heights $(E, N, H)$.

In all the above cases (Equations 5, 7, and 8), one can expect some problems. First, the transformation from orthometric to ellipsoidal heights — denoted as $OH \rightarrow EH$ in Figure 2 and Figure 3 — may not be known; this will be discussed in the next section. Second, the transformation from the $S$ system to the local system $L$ may also be unknown or scarcely known: if this is so, then two scenarios can be considered for the determination of the transformation, depending on whether the satellite system and the $U$ system coincide. Afterwards, the transformation parameters can be
used either before the adjustment or in the adjustment, as initial approximations and/or as weighted pseudo-observations.

If the S and the U systems coincide — having parallel axes and no scale difference, as happens to be the case with the NAD83 and WGS84 systems — then \( \mu = 0 \); \( R \) is either the unit matrix (Equation 8 model) or \( R = M(\lambda, \phi) \) where \( (\lambda, \phi) \) are the geodetic coordinates of \( O \) in the \( U \) reference system (Equations 5 and 7 models): \( X \) can be computed as the difference between the geocentric coordinates of \( O \) in the \( U \) system and those of the reference stationary receiver in the \( S \) system.

If the \( S \) system and the \( U \) system do not coincide, the datum transformation parameters must be available beforehand or be determined in an independent survey. However, if the transformation parameters are to be estimated in the
Table 2. Relative Errors of a Global Geoid Model (Units are cm).

<table>
<thead>
<tr>
<th>3'</th>
<th>6'</th>
<th>9'</th>
<th>12'</th>
<th>15'</th>
<th>18'</th>
<th>21'</th>
<th>24'</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>11</td>
<td>20</td>
<td>29</td>
<td>38</td>
<td>45</td>
<td>53</td>
<td>60</td>
<td>66</td>
</tr>
</tbody>
</table>

\( \sigma_N \) for normal terrain
\( \sigma_M \) for mountainous terrain

where \( h \) is the height in arc minutes, \( \sigma_N \) is the standard error of the normal terrain, and \( \sigma_M \) is the standard error of the mountainous terrain.

Adjustment, the procedure described above remains valid for computing initial approximations for \( (X, \mu, R) \).

Finally, but equally important, note that linear components of small discrepancies resulting from local control network distortions or from other sources will be absorbed by the drift parameters (see next section for a related discussion).

It goes without saying that in either of the approaches, the so-called “Earth curvature correction” for image observations must not be applied.

Ellipsoidal and Orthometric Heights

Orthometric heights \( (h) \), the heights used in surveying and mapping, are referred to the geoid, an equipotential surface mean sea level. Orthometric heights have a physical meaning: their differences tell us which way water flows; they lack a geometric meaning. Ellipsoidal heights \( (N) \), the heights given by and to be used with GPS techniques, exhibit just the opposite properties: they are only of a geometric nature.

Geoid undulations \( (N) \), which for practical purposes can be considered as the ellipsoidal heights of the geoid points, are instrumental in relating orthometric and ellipsoidal heights through the simple formula

\[
h = N + H. \tag{10}
\]

Correct use of any of the observation Equations 5, 6, 7, and 8 requires that control point heights be introduced as ellipsoidal heights and that adjusted heights be transformed back to orthometric heights for further use.

The whole transformation is a simple task provided that a geoid model yielding \( N \) is available. In many countries, high-resolution geoid models have been recently computed. They are usually available in gridded form together with field extraction and grid interpolation software. Typical accuracies of these high-resolution geoid models are of a few parts per million of the distance between points and can also be more accurate — in any case, much higher — than the accuracy of orthometric point determination. The introduced errors am, therefore, be disregarded.

Wherever a high-resolution geoid is not available, a global, world-wide model can be used. A set of global determinations which has become very popular is the family of GPS models (Department of Geodetic Science and Surveying, The Ohio State University; Rapp et al., 1991). An OSU model can be obtained directly from its author, R. H. Rapp (in terms of the coefficients of a spherical harmonic expansion of the gravitational potential, in the WGS84 system) or from some national geodetic agencies (sometimes in a more convenient gridded form and already transformed to the particular national reference system). A welcome development in this respect is that some GPS receiver and software manufacturers have these models embedded in their software.

In the context of this paper, the question is: "How much error is left by using a global model instead of a high-resolution one?" The other question regarding the error committed by using no geoid at all maybe misleading because global models do exist and their use is simple. See also Schwarz and Siders (1993) for a related discussion.

Answering the above question is not easy: though the spatial resolution of the current geoid models is 0.5' (about 55 km) and the average standard error \( \sigma_N \) is about 0.5 m, in areas with poor gravity coverage \( \sigma_N \) can build up to 2 to 3 m; equally, it can be reduced or enlarged by a factor of 2 to 3 in flat or mountainous terrain, respectively. At this point, however, the reader with no high-resolution geoid model at hand should not be discouraged: there are two factors, at least, which will help him or her to do much better than the above \( \sigma_N \) figures may seem to indicate.

First, because precise height determination by GPS is performed in differential mode, in practice, instead of Equation 10,

\[
\Delta h = \Delta N + \Delta H \tag{11}
\]

will be used, where \( \Delta h \) and \( \Delta N \) are ellipsoidal and orthometric height differences and where \( \Delta N \) are geoidal undulation differences, respectively. Therefore, we are interested in \( \sigma_{\Delta N} \) rather than in \( \sigma_N \). Because geoid undulations between close points are highly correlated, \( \sigma_{\Delta N} \) is much smaller than \( \sigma_N \). In other words, systematic errors affecting \( N \) to a large extent cancel out when computing undulation differences \( \Delta N \).

In Table 2, empirical estimations of \( \sigma_{\Delta N} \) are shown for several distances and for two terrain scenarios. They have been computed by comparing the OSU91A model with the high-resolution geoid model of Catalonia, which is assumed to be error-free.

Second, if datum transfer parameters are estimated in the block adjustment, they will account for an eventual non-modeled local geoid slope. If datum transfer parameters are either non-estimable or kept fixed, but drift parameters are used, the latter will account for a non-modeled local slope as well. If drift parameters per strip are used, then there will be the possibility of correcting non-modeled bilinear features. From a theoretical point of view, it might be slightly better to keep an independently well-determined datum transformation fixed and to estimate the drift parameters, because otherwise possible big errors in geoid undulations could have an unwanted effect on a global scale factor. In either case, the figures in Table 2 may be smaller by a factor of 3 or more.

Note that the problems discussed in the last two sections are general ones affecting not only photogrammetrists but the whole surveying community when integrating GPS surveys into the existing conventional control networks (see, for instance, Leick, (1980)).

The GPS Antenna Offset

Under operational flight conditions, the navigator-photographer controls the camera to keep the correct overlap between images. In general, the determination of the offset is not critical. However, for photogrammetric network densification and for large scale mapping, the situation may change; consider, for instance, a low altitude flight with unfavorable wind conditions where strips are flown alternatively in opposite directions. If, as the results of recent investigations (Cannon et al., 1992a; Cannon et al., 1992b; Schade, 1992) seem to indicate, a single drift parameter set will suffice in the future, then it will be necessary to record the antenna offset, or any equivalent set of values, at each exposure moment. Because metric cameras are operated by the navigator-photographer through remote control systems and/or automatically, information about the offset does exist and can be output in digital form.

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In experimental airborne GPS tests at the IIO, the offset was measured after the flight mission because the camera was operated in a locked-down mode. In the future, one will have to take into account on having some kind of information coming from the camera system and plan on measuring the antenna offset for several camera position readings. The information collected will constitute a 3D grid of calibrated offset vectors. Actual in-flight offset vectors will be interpolated from the grid. With this approach, the eccentricity vector $X_k$ will no longer be an unknown, but will be auxiliary data of the observation $x_k$.

However, if the above time varying offset is not available and accuracy requirements are high, a set of drift parameters per strip will suffice in most cases.

**Software Design Considerations**

A conclusion which can be drawn from the preceding discussion is that a software system for GPS-supported aerial triangulation has to incorporate a set of basic geometric transformation modules (see Figures 2 and 3) which include the interpolation of geodetic heights from global and high-resolution local geoid models.

Ideally, the block adjustment program should implement the models corresponding to Equations 5, 6, 7, and 8 so that, depending on the situation, it best fits the needs of the user. Programs with this capacity are available, and their implementation in already existing software should be but a minor software engineering problem. An additional module which at the very least allows for the adjustment of small GPS terrestrial networks and for the determination of datum transformation parameters between point fields is also recommended. In principle, a single "network adjustment software engine" could deal with any adjustment task so that advantage can be taken of the investment made both in development from the point of view of the software manufacturer — and in learning — from that of the user. See, for instance, Colomina et al. (1992) and the pioneering development in Elsasser (1983).

Although GPS phase processing is not the topic of this paper, it ought to be pointed out that a software system for GPS aerial triangulation should not necessarily restrict itself to purely photogrammetric aspects. It should either cover or integrate — through standardized data file or database systems — all steps from GPS phase processing to flexible block adjustment through the required intermediate transformations; after all, aerial triangulation systems are nothing more than point determination and sensor orientation systems.

**Concluding Remarks**

The analytics of GPS-supported aerial triangulation and particularly the coordinate transformations involved are basic geodetic concepts; the goal of this paper was just to review them from an operational point of view in the context of modern aerial triangulation with GPS aerial control. Nevertheless, there are three aspects which should be dealt with carefully when moving from conventional ground control to GPS aerial control: (1) the consistent determination of the GPS antenna eccentricity; (2) the datum transformation between the satellite reference system, $S$, and the systems $U$ and $L$; and (3) the transformation between ellipsoidal and orthometric height systems. Driven by the need to test the intrinsic properties of GPS kinematic positioning, most of the experiments conducted so far have bypassed the former issues by keeping the camera fixed and by working in test areas with local Cartesian coordinates determined by GPS terrestrial surveys.

For GPS-supported aerial triangulation to become truly operational, the above questions have to be tackled not case by case but with an overall policy. Metric camera manufacturers are mostly responsible for point (1) above; the information required certainly exists within their systems and the ongoing developments in stabilized mounts are a good opportunity. National geodetic and mapping agencies have to provide datum transformation parameters, information on local deformations of control networks, and geoid determinations; this solves the data aspect of points (2) and (3). Software manufacturers have to provide integrated solutions running from GPS phase observation processing to photogrammetric network adjustment, including all the coordinate transformation and ntapx projection modules required.

**References**


